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The Making of Metaheuristic Growth Theory: Key Ingredients, Math Formulas, and Empirical Tests*

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Abstract

With an initial cut into the Lucas puzzle, this paper develops a new theory, metaheuristic growth theory, to explain variations and variability of economic performance across different countries in the world by conjugating inductive historical reason with deductive mathematical reason. Highly mathematical in formulation yet intrinsically embedded in reality, with the kernel of the theoretical framework linking wealth with energy ultimately constrained by the physical laws of thermodynamics, the new model represents a paradigm shift from the canonical model of classical or neoclassical economics. It posits: in the real world of non-linear, non-convex, multipurpose, and complex ecosystems, growth is a function – probabilistic rather than deterministic – of a causal nexus of increasing depth and decreasing visibility – physical, institutional, motivational, and ideational. High in dimensionality yet equally rigorous if not more, the metaheuristic growth theory is greater in both predictive and prescriptive power than the old "static" and "lifeless" model of neoclassical economics.

Keywords

Lucas puzzle, Abductive approach, Theoretical synthesis, Mathematical modelling, Empirical observations.

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The supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.

Albert Einstein

1. Introduction

Why have some nations developed whereas others stagnated (Lucas 1988; 1990)? To answer this profound question generally known as the "Lucas paradox" or "Lucas puzzle", metaheuristic growth theory represents a new theoretical framework, or a "paradigm change" (Kuhn 2012), if one may, from the "old" model of neoclassical economics. It does so *not* by rejecting mathematics as powerful tools for science, but by deepening our understanding of its philosophical underpinnings, especially the fusion of geometry, algebra and topology, which, I submit, is the umbilical cord connecting what ancient Greek philosophers called *rhema* (instant truth) and *logos* (constant truth). In our age, the interface between the two realms of truth, or the "connectomics" (Sporns et al. 2005; William 2011), if one may, is embodied ever so subtly by the beauty of digital twins.

Now armed with an updated version of cognitive toolkit, we can launch higher-dimensional investigations in our search for a simple, but not simplistic, organizational framework of diverse phenomena in the real world of economic activity in non-linear, non-convex, multipurpose, and complex ecosystems. Let me say at the outset: It is by increasing dimensionality and at the same time, imposing the classical limit of the physical laws of thermodynamics as the critical kernel of our theoretical framework that our new model is both rigorous and relevant (i.e., relevant to real world practice: I have here policy practitioners in mind and will return to this point in conclusions) and holds more explanatory power than the canonical model of classical or neoclassical economics. And lest we forget, our model is also highly falsifiable, a critical criterion of research being scientific (Popper 1959).

The rest of the paper is organized as follows: In section 2, I lay out the key ingredients of the metaheuristic growth theory, making comparisons and contrasts with the classical or neoclassical economics, including underlying philosophical thinking, where relevant. In section 3, I formalize our new theory mathematically, including quantum simulation in decoherence at the macroscopic level of institutional configurations – a sort of hyped-up matrix or triplet (cf. Brenner 1957) between the state, markets, and society in search of regional and/or global optima. In section 4, I show how our new model works with empirical evidence, including empirical demonstration of the link between energy and wealth as the kernel of our theoretical framework. Conclusion follows in section 5, with a recap of the underlying syntax of our new theory, including a brief review of how I get here by looking back at history.

2. Key Ingredients

The limits of our language are the limits of our world, as Ludwig Wittgenstein (1922) would remind us. Let me start by looking at the term *metaheuristic*.

Etymologically, *meta* comes from an ancient Greek prefix μετα-, meaning ever transcending and overarching, as in metaphysics; *heuristic*, another Greek word εὑρίσκω, means finding resolutions via techniques that are inductive, indeterministic, historical, experimental, or "rules of thumb". Put together, *meta+heuristic* growth theory means a dynamic and asymptotical process – embracing both *logos* (constant truth) and *rhema* (instant truth) (cf. Guthrie 1971)[1] – in a logarithmic spiral of human progress from what Immanuel Kant (1781) called *synthetic a posteriori* (inductive historical reason) to *synthetic a priori* (deductive mathematical reason). Epistemologically, these two sets of terms, i.e., logos and rhema, and synthetic a priori and synthetic a posteriori, conceivably cover the whole spectrum $\epsilon(0,1)$ from heuristic to ontological modelling. By the way, cross-culturally, the tension between rhema and logos, or between *instant truth* and *constant truth*, is deeply shared by the notion of *Tao* in Taoism of Chinese indigenous religio-philosophical tradition, which postulates that «道可道非常道» or «The *Tao* (The Way) that can be spoken of is no longer the *constant Tao*» (cf. Creel 1982; Thompson 2003).

For simplicity, Gottfried Leibniz (1679; 1966) believed that the whole universe can be reduced to two things: 0 and 1. To that, for elegance, let me add two Forms, i.e., O (circle) and Δ (triangle), such that we have the forever subtle interface between truth and reality or Euclidean and fractal geometries. Now with these few simple and elegant *things*[2] or *symbols* as the most fundamental building blocks bridging the mind and the body – or what philosophers would call the mind-body problems (cf. Bunge 2014), we have entered the digital age, embodied ever so subtly by the beauty of digital twins (cf. Georgiev 2020).

Seen in this light, one may take neoclassical economics exemplified by the Arrow-Debreu model (Arrow & Debreu 1954) as a *meta*-theory without the *heuristic* part. It is by assuming away a content-rich context of the reality that the model "mathematically" proved the «existence of the Walrasian general equilibrium of competitive markets» (Walras 1874).[3] This is admirable as a counterfactual thought experiment, but by itself it is grossly incomplete. Indeed, Galileo Galilei did the same thing when, as an initial step of investigation, he assumed away the air; but subsequently he had to bring the air back in order to figure out the secret mechanics of falling bodies, including frictions.

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[1] Following Wittgenstein, at least language-wise, one may argue that the origin of the universe begins with the combination of logos and rhema hidden in the 5th postulate of Euclidean geometry where false vacuum exists. Prior to that, there is no space-time, where, as I submit, Einstein believed that "god does not play dice". After that, a simple program of fusion between logos and rhema starts to generate uncertainty (Heisenberg 1927) and complexity (cf. Vermur 1978), which eventually give rise to the cosmos, life, and human consciousness. In Chinese philosophy – the Chinese version of Buddhism in particular – the concept of *se ji kong* (色即空) comes close to the interface between rhema and logos, which may be mathematically expressed as $\infty \rightarrow 0$, a fascinating topic that can provide insights into the deep principles of connectome between East and West. But we do not have space to explore and elaborate here in this paper.

[2] Recall Martin Heidegger's philosophical question: "What is a thing?" Heidegger (1970) makes a distinction between things and objects in that an object becomes a thing when it can no longer serve its common function.

[3] The Arrow-Debreu model posits that under certain assumptions, i.e., convex preferences, perfect competition, and demand independence, there must be a set of prices such that aggregate supplies equal aggregate demands for every commodity in the economy.

Thus, instead of «throwing the bathwater out with the baby», so to speak, our challenge is to bring in the context, or the *field* in modern physics, such that our new model has both *form* and *content*. Indeed, as exemplified by Maxwell's equations (1873), the concept of field is essential to modern physics (i.e., relativity and quantum mechanics). It all began as a courageous scientific imagination that was needed to realize fully that not the behavior of bodies, but the behavior of something between them, i.e., the field, is essential for ordering and understanding events. With the field, the background for all events is no longer 1-dimensional time, nor 3-dimensional space, but 4-dimensional time-space continuum. And going into the intension of bodies, quantum theory formulates probability laws that govern *crowds* and not *individuals* in Hilbert space – an imaginative vector space that mathematically features dot inner product ≤1.[4]

Inspired, metaheuristic growth theory: (a) views economic growth as a logarithmic spiral of human progress that is historical (cf. Toynbee 1934, 1939, 1954, 1961; Gadamer 2016) and social-biological (Wilson 1975); (b) strives to bridge the gap between truth and reality by conjugating deductive, universal, mathematical approach with inductive, particular, historical approach; and (c) enclose the whole theoretical framework by imposing the classical limit of the physical laws of thermodynamics, thus linking energy with wealth as the critical kernel of our theoretical framework.[5] For, consistent with Einstein's mass-energy equation $E=mc^2$, in a final analysis, wealth is energy in stock or in flow;[6] human beings survive and thrive in anti-entropic struggles in exosomatic evolution (cf. Lotka 1922); and in the process, structural dynamics is compatible with evolutionary change (Scazzieri 2018).[7]

Indeed, as the "meta" part of the growth theory suggests, it is the kind of idea – a higher-level, overarching conceptual framework – that puts all other ideas or all previous theories of value in a unified perspective, or a general theory, if you may.

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[4] Hilbert space is named after David Hilbert (192; 2004). In mathematics, it is a vector space equipped with an inner product that induces a distance function for which the space is a complete metric space. As such, it allows methods of linear algebra and calculus to be generalized from a Euclidean space of finite dimensions to one of infinite dimensions.

[5] Specifically, the law 0 of thermodynamics says that two systems in equilibrium with a third system are in thermal equilibrium with each other; the law 1 says that energy can change forms but is neither created nor destroyed; the law 2 says that entropy of an isolated system always increases; and the law 3 says that entropy of a system approaches a constant as temperature approaches absolute zero (cf. Atkins 2007).

[6] Existing in 4 fundamental states, i.e., solids, liquids, gases, and plasmas (cf. Goodstein 1985), matter is the material substance that constitutes the observable universe and, together with energy, forms the basis of all objective phenomena. Energy, in physics, is the capacity for doing work or producing heat. It may exist in potential, kinetic, thermal, electrical, chemical, nuclear, or other various forms (Jaffe 2018). Consistent with $E=mc^2$, the laws of thermodynamic describe the relationships between matter and energy under different conditions.

[7] A stylized mathematical formulation and demonstration for that is to tip a circle into an ellipse within a clone, such that one focal point of the circle becomes two focal points of the ellipse, the distance of which determines the *modular* elliptic curvature and associated stress levels. (Note in particular here, by *modular*, it indicates that it is no longer a classical approach). The larger is the distance, the more stressful the spiraling-up change is, and *vice versa*. Zero distance between two focal points means repetition of history, that is, no evolutionary change or structural dynamics.

As such, straddling a wide range of knowledge in arts and sciences, our new model is transdisciplinary, holistic, and abductive, that is, a combination of both deductive and inductive reasoning (Peirce 1903).[8] And yet, in spite of its multiplicity, it can be encapsulated and expressed in simple math formulas, including quantum stimulation, which philosophically is rooted in the fusion of logicism (e.g., Euclidean geometry), formalism (algebra), and intuitionism (topology). Take a look at $\int |\psi|^2 dq = 1$.[9] What one can see here is not just *logic* but also *intuition*. Little wonder Paul Dirac famously said: «A physical law must possess mathematical beauty.» Here I would take beauty as truth in mathematical intuition, as against the classical logic of *tertium on datur*, i.e., the law of excluded middle ($p \vee \sim p$) (cf. Whitehead & Russel 1910), which has ultimately turned the canonical model of neoclassical economics into a "static" and "lifeless" theory.

Thus, axiomatically or foundationally, the metaheuristic growth theory is different from classical or neoclassical economics in that it embraces *homo sensus sapiens* rather than *homo economicus*. In other words, human beings both reason and feel (cf. Tversky 1981; Simon 1957; Shiller 2000), and between sensitivity and rationality, we are *analog* rather than *digital*. As such, our theory is inclusive of both the *rational* and the *intuitive* parts of the human mind – the fountain of human creativity: inspiration, imagination, invention, and discovery. And in this connection (cf. Mokyr 1990; 1998; 1999), innovations of various sorts – including institutional technologies such as markets and the rule of law – are only its derivatives (cf. Nelson 1992). They are products of human exosomatic evolution.

Albert Einstein is widely quoted as saying: «The intuitive mind is a sacred gift and the rational mind is a faithful servant» (Samples 1976). Mindful that the "old" model of classical or neoclassical economics has created a culture that honors only the servant and has forgotten the gift, we strive to correct that lopsided mistake by taking a broader and deeper perspective, higher in dimensionality.

Specifically, I take neoclassical economics only as a *benchmark* devise (albeit unrealistic), deploy it only *heuristically*, but move on to build on it with four more building blocks: (a) *engine*, (b) *field*, (c) *time*, and (d) *fuel*. By *engine*, I mean the creative human mind. By *field*, I mean the institutional/cultural context – a sort of hyped-up matrix of the state (governance), markets, and society (social conventions) amenable to topological or group-theoretic analysis (cf. Carlsson 2009). By *time*, I mean a historical line, biological rather than physical, linking past, present, and future, and sensitive to initial conditions and path-dependence. By *fuel*, I mean energy, defined as the capacity for doing work or producing heat. Accordingly, by imposing the classical limit of the physical laws of thermodynamics, I cap the whole model off, as it were, within Hilbert space which mathematically features dot inner product, such that our theoretical framework is self-enclosed and complete – at least *physically* if not *mathematically* – like a set of the Russian Matryoshka dolls that fit one into the other.

[8] Deductive reasoning starts with general rules and arrives at specific conclusions that are necessarily true; and inductive reasoning starts with specific observations and arrives at general conclusions that may or may not be true. By comparison, abductive reasoning, by combining both, starts with incomplete observations and arrives at best predictions which may be *asymptotically* but *not necessarily* true.

[9] It is the generalized equation of quantum mechanics, also known as the *normalization condition* of wave functions.

The metaheuristic growth theory is thus high-dimensional, holistic, abductive, dynamic, and holographic. It posits: in the world of non-linear, non-convex, multi-objective, and complex ecosystems with risk and uncertainty (cf. Bernoulli 1738; Knight 1931; Allais 1953; Kahneman & Tversky 1979; Zimmer 1983; Loewenstein et al. 2001), economic growth/development (indicated by GDP per capita, following Kuznets (1966)) is a function – *probabilistic rather than deterministic* – of a 3-tiered causal nexus of *increasing depth* and *decreasing visibility*: *physical*, *institutional*, and *ideational*. And the whole overarching model is ultimately constrained by the classical limit of the physical laws of thermodynamics.

In terms of increasing depth and decreasing visibility, recall here the memorable phrase of "invisible hand" by Adam Smith (1976) when he referred to the power of markets. Taking a step further, I submit that the power of human ideas is deeper – it is the invisible hand of the invisible hand. But then, the human brain has to be driven by energy. And energy, defined in physics, is a conserved quantity, measured in J (joule), that can be converted in form but not created or destroyed. All living organisms take in and release energy constantly. Human civilization requires energy to function, which it gets from energy resources such as fossil and non-fossil fuels. And the Earth's climate and ecosystems are driven by the energy that the planet receives from the Sun.

Not coincidentally, Douglas North (1973) was deep and perceptive when he said that «the factors we have listed (innovation, economies of scale, education, and capital accumulation etc.) are not causes of growth; *they are growth*». He was actually criticizing the superficiality of the mainstream economics, such as the Solow growth accounting model (Solow 1956) and its variants (Mankiew, Romer & Weil 1992),[10] for focusing only on *proximate* causes while ignoring *deeper* and *fundamental* causes. And on his part, the Irish poet and playwriter Oscar Wilde (2006) said somewhat paradoxically: «It is only shallow people who do not judge by appearances. The true mystery of the world is the visible not the invisible». And Galileo Galilei, a member of the Lincei Academy in Italy since 1611, would sign his scientific works as "Lincean", lynx now being the logo of the Accademia Nazionale dei Lincei – symbolizing the ability to penetrate into the invisible realms to unlock the secrets of Nature (Quadrio-Curizio 2018).

3. Math Formulas

Carl F. Gauss, known as *Princeps mathematicorum* (the prince of mathematics), famously said: «Mathematics is the queen of science, and arithmetic the queen of mathematics» (cf. Hall 1970). If, as philosophers of science would do, the superiority of competing theories is to be judged by the criteria of simplicity, consistency, accuracy, completeness/scope, and fruitfulness, mathematics must be the language of choice. But even in the pristine world of pure mathematics, human beings so far have failed to achieve *consistency* and *completeness* simultaneously, as per Gödel's incompleteness theorems (cf. Smullyan 1992).

[10] Mankiew et al., added savings rate, population growth rate, and initial level of labor productivity to Solow's growth accounting model which focuses on capital accumulation, physical and human. But both models remain a-institutional and a-historical.

And back to the real world where fractal geometry rather than Euclidean geometry rules, strictly speaking, all mathematical models are approximations to reality, as it takes both universal forms and particular contents to make predictions. The art of science is to make rational approximations and the rigor comes in specifying as precisely as possible the circumstances where approximations break down in particular limits. Paul Dirac (1930) said it well: «I understand what an equation means if I have a way of figuring out the characteristics of its solution without actually solving it». Note that he was a theoretical physicist rather than a pure mathematician.

Asymptotic probability is the key.

Based on the foregoing, guided by the principle of parsimony, or the so-called Ockham's razor which states that *pluralitas non est ponenda sine necessitate* (plurality should not be posited without necessity), and philosophically sensitive to the tension between proof-as-algorithm/para-consistency (cf. Brouwer 1952) and proof-as-consistency = existence (based on classical logic, i.e., *tertium non datur* (law of excluded middle), set theory and number theory) (cf. Hilbert 1925; Posy 1998),[11] I formulate the metaheuristic growth theory mathematically as follows:

$$G_t \approx \int \left\{ \left[\frac{K_N^{\alpha} \cdot K_N^{\beta} \cdot K_P^{\gamma}}{(1 - \Psi K_i^{(v,h,s)})} \right] /(1 - \delta_I) \right\} dt \, *$$

$$\alpha + \beta + \gamma = 1$$

$$\Psi K_i^{(v,h,s)} \in (0,1)$$

$$\delta_I \in (0,1)^{12}$$

where: G stands for economic growth, t for time, and ≈ for approximation. K_N denotes *natural capital*, K_H *human capital*, and K_P *physical capital* (which may include data today). Together they refer to *physical causes* of growth. K_i stands for *institutional capital*, which in turn has three dimensions: K_i^v (institutional capital-vertical) denotes *state*, K_i^h (institutional capital-horizontal) *market*, K_i^s (institutional capital-social) *society*. Together they refer to *institutional causes*. δ_I denotes the idea gap, taken here as *ideational causes*. The notation $\alpha+\beta+\gamma=1$ implies constant return to scale; as a corollary, if $\alpha+\beta+\gamma<1$, return to scale decreases; if $\alpha+\beta+\gamma>1$, return to scale increases. Modulus $\{0<\Psi K_{\mathrm{i}}^{(v,h,s)}<1\}$ denotes quantum simulation of institutional context in 4-D continuum, i.e., the state (with priority utility function of power), markets (wealth), and society (love), plus time (history). * denotes classical limit of thermodynamics, thus making a distinction between *pure* and *applied* mathematics.

[11] Following the Arrow-Debreu proof (1954; 1959), the prevailing mode of mathematical formalism as practiced in the economics profession has, consciously or not, subscribed to the theme of proof-as-consistency=existence, crowding out any room for entrepreneurship facing Diophantine decision problems. Note in particular, without numerical content, the Arrow-Debreu proof (1959), while it may be logically tight, ultimately is an incomputable general equilibrium, that is, not a computable general equilibrium (CGE) model – an intellectual blind spot that one must be aware and avoid.

[12] The expression, i.e, $\delta_I \in (0,1)$, ultimately reflects the extremely creative work of the human mind expressed on one hand by "Euler's god equation" – $e^{\pi i} + 1 = 0$ (*pure mathematics*), and on the other hand, by what Erwin Schrödinger said about *quantum physics* – «quantum physics thus reveals a basic *oneness* of the universe» (cf. Schrödinger 1996).

Note in particular that in bridging truth with reality in mathematical modelling, one ultimately has to turn a pure math equation into an equation of mechanics. How to do that in our case? Hermann Weyl, famous for his ability to link pure mathematics with theoretical physics and unite previously unrelated subjects, had this advice: «My work has always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful». And in his *Die Idee der Riemannschen Fläche* (Weyl 1913, 2009) (*The Concept of a Riemann Surface*), Weyl (1997) created a new branch of mathematics by uniting function theory and geometry, thus opening up the modern synoptic view of analysis, geometry, and topology.

Inspired and drawing upon the deep insight of Einstein's energy-mass equation $E=mc^2$, my strategy, or deux ex machina, if you may, is to impose the classical limit of the physical laws of thermodynamics, denoted by * in our equation, thus making a distinction between *pure* and *applied* mathematics, and from there I let all key variables parameterized in our model operate iteratively and asymptotically toward global optima (defined as contingent but not absolute equilibrium),[13] as it were, in Hilbert space that features dot inner product. And then let us see who gets ahead and who is behind in the Euler-Lagrange equation that can calculate maxima and minima of a complex system with multiple coordinates (cf. Artken 1985; Dirac 1933). Generally speaking, a linear approximation of non-linear, non-convex, multipurpose, and complex ecosystems in the vicinity of global optima as a critical benchmark devise is instrumental in studying the qualitative behavior of the complex ecosystems as time approaches infinity.

Metaheuristic growth theory so formulated, several salient features stand out and are worth noting:

(1) As a simple, but not simplistic, organizational framework of diverse phenomena of economic activity in complex non-linear ecosystems, the model is highly constructible and scalable – up and down all the way from, or even within, the business level to the global level, and vice versa, depending on the unity of analysis one is looking at, where indeterminacy and ambiguity underpin perfectly rational decision-making, and the solvability of Diophantine decision problems lies in asymptotic probability.

(2) The model is neither inductive nor deductive singularly but abductive as a whole, combining both historical reasoning and mathematical reasoning, as the father of existentialist philosophy Søren Kierkegaard (1849) reminded us: «Life can only be understood backwards; but it must be lived forwards». Indeed, reaturing asymptotic probability, the model is amenable to Markov decision processes (cf. Gagniue 2017) and Bayesian approach (Bayes 1763); but ultimately all has to be constrained by the classical limit of the physical laws of thermodynamics, or the «hard as against the soft budget constraints», so to speak, in the language of economics (Kornai 1980).

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[13] What it means is that, while our mathematical equation is approximately symmetrical, hence the symbol ≈, strictly speaking, the principle of conservation of parity, or the so-called *P*-symmetry formalized by Eugen Wigner (1927; 1959), does not hold at the very bottom of our model either, as per the law 2 of thermodynamics. It has experimental implications, when we say «wealth is energy, in stock or in flow». Specifically, R^2 can never be 100% in any regression analyses between energy and wealth.

(3) The model is high-dimensional and transdisciplinary.[14] The modulus $\{0<\Psi K_i^{(v,h,s)}<1\}$ denotes quantum simulation in decoherence at macroscopic level of institutional context in a 4-D continuum – a sort of hyped-up matrix of the state, markets, and society, plus time (history) in search of regional and/or global optima; correspondingly, just as in topological data analysis, it has to involve multiple domains of knowledge (e.g., political science (Hobbes 2006; cf. Turchin 2003), including its subfield, international relations (cf. Keohane 2005; Jervis 1993), economics, sociology/anthropology, and history), so that one becomes sensitive to patterns, if any, of persistent homology (cf. Cagliari et al. 2011), and can inform persistent modules in light of kernel-based statistics (cf. Shaw-Taylor 2004). Denoted by *, the kernel of our model is the link between wealth and energy, as per thermodynamics.

(4) With a high-dimensional field built into it, the model is holographical. As Figure 1 illustrates *visually*, as if in the tunnel of time where Markovian process rules (cf. Shapley 1953; Bellman 1957; Puterman 1978; Shwartz 2002), as one (or firm *X*) travels along the historical line forward or backward, one can feel the perennial tensions between universals and particulars, with all sorts of hysteresis due to initial conditions and path-dependence (cf. Liebowitz 1995; Vergne 2010), as well as risk and uncertainty.[15] As such, the role of entrepreneurship becomes salient – sources of what Joseph Schumpeter (1942) has called «the perennial gales of creative destruction». Further still, as one travels increasingly towards the tip of the cone, *range* measures have to converge increasingly to *point* measures via elliptic curvatures; correspondingly, energy level goes up,[16] and so does heat (Incropera 2012).[17] This has deep implications for us to take the near parity of "energy is wealth" as the kernel of our multi-layered theoretical framework.

[14] By comparison, being low-dimensional (less than 3D), the relative merits or rather shortfalls of some of the most celebrated economic models would become transparent immediately. To illustrate, Smith's model (1976) is Euclidean 2-dimensional (state orthogonal to market), with the vector pointing one way towards the market (until the market fails in providing what Smith calls «three duties of great importance», i.e., «security, social justice, and public works». Coase's model (1937) is also Euclidean 2-dimensional (hierarchy and market), but with the vector moving up or down diagonally, thus raising the critical question of where to draw the line between hierarchy (by extension state) and market in order to minimize transactions costs. Ostrom's model (2007) is potentially non-Euclidean 3-dimensional, as the model brings in the social dimension of community to fill up the lacuna between state and market in iterative games. However, while it adds a time dimension in iterative gaming, it omits state and market altogether. As such, its explanatory power is limited in impersonal and arm's length transactions, especially when we consider interconnectivity in complex ecosystems in a global setting full of risks and uncertainties.

[15] Not coincidentally, in the field of visual arts, the Spanish painter Pablo Picasso said pointedly: «Painting isn't an aesthetic operation. It is a form of magic designed as a mediator between this strange hostile world and us» (Gilot 1965).

[16] It can be formulated mathematically as $\Delta E=hf$, or for the sake of simplicity in analysis, $E_1 \rightarrow E_2 \rightarrow E_3 = hf$ in different levels of energy (*E* denotes energy; *h* time; *f* frequency).

[17] In the four-fundamental states of matter, i.e., solid, liquid, gas and plasma, imagine here nuclear fission or fusion in plasma.

(5) Finally, in the new model (again as Figure I illustrates), the "old" model of neoclassical economics (e.g., Cobb-Douglas production function) (cf. Douglas 1976) no longer operates "in vacuum". Indeed, as if in a shift of Gestalt (cf. Murray 1995), one has to change the way that we look at mainstream economics. The key is network effects of symbiosis (cf. Paracer & Ahmadjian 2000; Lenski et al. 2003) To illustrate, variables K_N, K_H, and/or K_P can no longer be viewed as exogenously given, but have to *interact* and *emerge* endogenously with K_i, hopefully with the right kind of public policies, for them *to become* (as against *to be*) comparative or competitive advantages in a global setting. In this context, leadership or entrepreneurship can be understood as culturally/institutionally shaped shapers with new visions, denoted as δ_I, i.e., the idea gap, in the new model – the most fundamental cause of all of economic growth/development.[18]

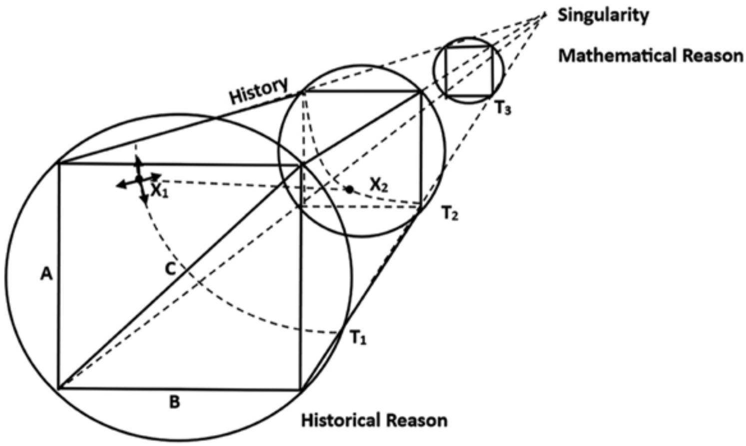

Fig. 1. Production function in holographic complex ecosystems (not drawn to proportions, for illustration only), where: *X* stands for firm, or a production function; *T* for historical time; *A* for state; *B* for market; *C* for society. 4 vectors indicate pro-state (pro-power), pro-market (pro-wealth), pro-globalism (pro-universalism), or pro-localism (pro-particularism) respectively. For illustration, $T_{1..2}$ indicates market-oriented reforms and opening-up in China in the past 40 years; and $X_{1...2}$ indicates that the institutional context has changed significantly from plan to market and it continues to evolve due to domestic and international pressures.

For many a brilliant mathematician like Gauss, *simplicity* and *elegance* are the touchstones of their work, and novel concepts, however strange they appear at first, tend to win out in the long run if they help to keep the subject simple and elegant (Gauss 1876; Winger 1925). As Gauss said analogously: «When one has constructed a fine building, the *scaffolding* should no longer be visible» (emphasis added).

[18] Even Karl Marx (1850), who believed in the "inexorable train of history", did not totally deny individual autonomy in history. He said that «men make their own history, but they do not make it just as they please; they do not make it under circumstances chosen by themselves, but under circumstances directly found, given, and transmitted from the past».

Inspired, we may consider treating K_H, K_P, and $K_i^{(v,h,s)}$ as intermediate variables – for they are only derivatives of human ideas, as I have argued earlier – and taking them out – along with all the problems of confounding and collinearity – from the convoluted causal nexus of economic growth. Assuming $\mathbb{E}[f(g(X_n))] \rightarrow \mathbb{E}[f(g(x))]$, i.e., continuous mapping regarding convergence or distribution in probability (cf. Kelley 1975), we may want to try to reduce and reformulate our growth equation simply as:

$$G \approx K_N / (1 - \delta_I),$$

$$\text{where } \delta_I \in (0,1)$$

Which says that operating within a self-enclosed system (i.e. within the classical limit of the physical laws of thermodynamics, in our case) as it were, in Hilbert vector space that features dot inner product, as *idea* becomes *ideal* (or perfect in Platonic Forms[19]), wealth becomes infinite, given K_N (energy, water, minerals[20]). Note as well, when idea becomes ideal, a *dot* becomes a *point*, and as a corollary, fractal geometry becomes Euclidean geometry. Note further, whereas a *dot* can be subject to real-world analysis in Hilbert space with inner product, a *point* cannot.[21] For, as per Euclidean geometry, *a point is that which has no part.*

Thus, mathematically so formulated in our equation to highlight the power of ideas $\delta_{I,}$ which is non-rivalrous, as against natural capital K_N, which is rivalrous, the metaheuristic growth theory is foundationally a model that features the *law of increasing returns* as against the *law of diminishing returns* of the classical or neoclassical economics, also known as the *law of diminishing marginal productivity* (cf. Brue 1993).

And consequently, keenly aware of the critical importance of both theory and experience, concerned about both individual freedom and social justice as crystalized by Isaiah Berlin's two (positive and negative) concepts of liberty (cf. Berlin 1952, 1952, 2000; Galipeau 1994; Crowder 2004) and recognizing the imperative of communicative (as against mere instrumental) rationality (Habermas 1987; cf. Dallmayr 1988), our approach to the growth equation can only increasingly and asymptotically turn from one of bottom-up, inductive, and historical rea-

[19] According to Plato, whereas Forms are unqualified perfection, physical object are always qualified and conditioned (Plato 2000; Dancy 2004).

[20] In the future, even when humans are able to build "artificial sun" by nuclear fusion (cf. Kramer 2011), thus having a limitless source of clean energy, minerals are still finite on Earth. Circular economy (cf. Iacovidou 2021) therefore is the way to go.

[21] Incidentally, it would be difficult to translate into Chinese. Both *dot* and *point* are the same word "点" (dian) in Chinese. Wittgenstein famously said: «whereof one cannot speak, thereof one must be silent». Indeed, I would submit, seeing the difference between a point and a dot and, even more crucially, between the extension and the intension of a dot, and subjecting a dot to further analysis in Hilbert space with inner product (cf. Hilbert 2004) would make us rethink the theoretical foundation of the neoclassical model of international trade (cf. Marshall 1879; 1930; Bhagwati 1983; Samuelson 1947). One thing is sure: in light of Hilbert space featuring do inner product, "comparative advantage" (cf. Richardo 1817) no longer contradicts but is compatible with "competitive advantage" (Porter 1985). *Cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.*

soning to one of top-down, deductive, and mathematical reasoning as a continuous process of graduation. For, for the majority of people, "existence precedes essence" (Sartre 1946, 1956; cf. Marx 1850; Lessing 1967). In the meantime, via a constructive paradigm of categories plus toposes (cf. McLarty 1995), I would take mathematical formalization (cf. Tarski 1987; Rautenberg 2010) as a process – i.e., *becoming* rather than *being* – of a constructive *predicate* calculus converging increasingly towards a constructive *propositional* calculus. And yes, this approach is largely consistent with the latest discoveries of neural science (Miller & Buschman 2007; cf. Mittal & Narayanan 2021).

Philosophically, the simple and elegant mathematical formulation above also has deep implications for us to explore the origins of the universe, life, and human consciousness. Anyway, highlighting the importance of theory, Einstein said: «it is our theory that determines what can be observed». Likewise, John Holland (1962; 1992; 2012), a prominent computer scientist and pioneer in complex adaptive systems research, had this advice to offer: «With theory, we can separate fundamental characteristics from fascinating idiosyncrasies and incidental features. Theory supplies landmarks and guideposts, and we begin to know what to observe and where to act».

4. Empirical Tests/Observations

Throughout its lengthy history, mathematics has taken its inspiration from two sources – the real world and the world of human imagination. Once stepping out of the Platonic shadowy world, mathematical models are always approximations to physical reality. For empirical tests of our model, methodologically, the Gaussian scheme of probabilities distribution with varying degrees of deviation or perturbation in conjunction with Hilbert space featuring dot inner product can serve as a powerful tool to structure real-world observations. Let us do four sets of empirical tests/observations of our model with respects to 1) China in global perspective; 2) the power of institutions; 3) the power of ideas; and 4) the link between energy and wealth as the underlying kernel of the metaheuristic growth theory.

1) China in global perspective

As I said above, in the real world of complex systems, to give structure to empirical observations, generally the Gaussian framework remains a powerful cognitive toolkit – at least as an initial bench mark devise (Fig. 2). But one should stand ready to adjust, revise, or refine abductively in light of empirical evidence, especially when the picture is a dynamic one. Specifically, in our case of economic growth, we may take the framework as a global schema (cf. Tarski 1933) of non-linear, non-convex, multipurpose, and complex ecosystems where localized clusters with relatively invariant features – physical, institutional, motivational, and ideational – exist in probability distribution but through *tâtonnement* they collectively converge towards 1, but never arrive at 1, because the whole system continues to evolve *ad infinitum*. Paradoxically, *quod optimum test est melius* (the best can be better), so to speak.

Mathematically, and indeed philosophically as well, what one gets here is a picture different from the Arrow-Debreu model which has reached the so-called «Kakutani fixed point»

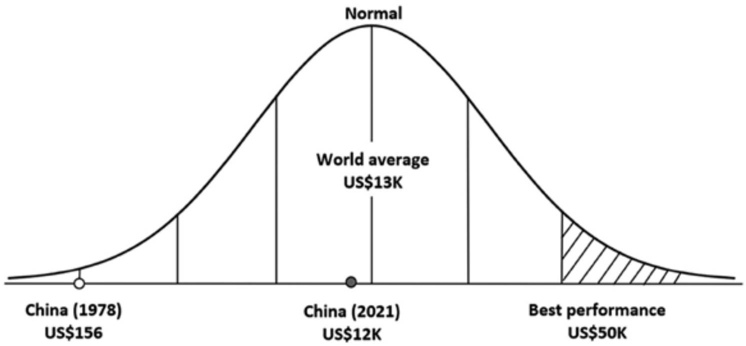

Fig. 2. China in global perspective (GDP per capita).

(Kakutani 1941). But, alas, in so doing, their model becomes "*lifeless*" and "*static*", leaving absolutely no room for what Schumpeter has called «the perennial gales of creative destruction». Thus, intellectually, one may take the not a Arrow-Debreu proof as a QED moment[22] in low-dimensional schema, but definitely not a Eureka moment in the real world of economic life in high-dimensional (n>3) schema.[23]

Viewed in the Gaussian framework in the real world of non-linear, non-convex, multipurpose, and complex ecosystems, the dramatic rise of China's economic performance – measured by GDP per capita, a key measure of *productivity*[24] – in the past 40 or so years represents a case of convergence rather than divergence in a global setting. Indeed, seen through the lens of our holographical model, it is by way of market-oriented reforms and opening-up – especially after China's entry into the WTO in 2001 – that China was able to move forward rapidly, as it were, in search of global optima, whence a learning-by-doing process (Arrow 1962; Fudenberg & Tirole 1983) intensified under conditions of globalization.

Empirical observations? The dramatic rise of Shenzhen as a special economic zone from a small fishing village to one of top 20 megacities in the world (measured by GDP) is but a case in point (Fu 2024). More systematical statistical evidence nationwide? China's GDP per

[22] QED is an abbreviation for the Latin phrase *quod erat demonstrandum* (that which is to be demonstrated) – a notation often placed at the end of a mathematical proof to indicate its completion.

[23] This would be entailed by Fermat's Last Theorem, which states that $x^n + y^n = z^n$, where x, y, z, and n are integers, has no non-zero solutions for $n > 2$ (cf. Cipra 1996). And in this connection, the "lifeless" and "static" model of neoclassical economics as exemplified by the Arrow-Debreu Proof can be saved by the mathematically more powerful modularity lifting technics used to prove the Fermat's Last Theorem (cf. Diamond 1995). The key is to raise dimensionality such that it opens up space *ad infinitum* for entrepreneurship in facing Diophantine decision problems to play a role. Indeed, as it were, breathing "life" into the "static" and "lifeless" Arrow-Debreu model of neoclassical economics, one may take it as another example of what Eugene Wigner (1960) has called «the unreasonable effectiveness of mathematics» (cf. Islami 2022) in economics.

[24] As Paul Krugman (1994) said: «Productivity isn't everything, but in the long run it is almost everything. A country's ability to improve its standard of living over time depends almost entirely on its ability to raise its output per worker».

capita was US$ 156 in 1978; it is about US$ 12,000 today, very close to the global "*norm*", which is about US$13,000. In the process of transformation from plan to market where, as I argued earlier, structural dynamics is compatible with evolutionary change,[25] China has become a powerhouse of manufacturing in the world, lifting over 700 million people out of poverty in the meantime.

In a broader historical perspective, however, this is not surprising. Indeed, taking the "advantage of backwardness", as well as "technological diffusion", (Gerschenkron 1962; cf. Nelson & Phelps 1966; Helpman 1993; Barro 1997) under conditions of globalization, China is returning to its historical heights. Some two hundred years ago, China had a population which was about 1/3 of the world's total, and it was producing over 1/3 of the world's total GDP (Maddison 2001). That was "*normal*", given the level of technology at the time. Today China's population is about 1/5 of the world's total, and it is producing about 18% of the world's total GDP. There is still room for further growth to get just "*normal*" in a global setting. Of course, this is by no means easy task, given the sheer size of China's population.

Having said that, however, for China, with 1.4 people, to move beyond the "*norm*" in the Gaussian scheme of probabilities distribution and nudge close to the global optima (about US$ 50,000. See the shaded area in Fig. 2) would be very difficult. For, that would require – besides natural resources constraints – playing a very different ball game, the name of which is no longer *imitation* at lower costs (roughly 65%) but *innovation* in all its forms (Mansfield 1981), together with the entire cultural/institutional milieu – political, economic, and social – to support it. But, as Max Weber (2011) saw it, any advice to a society to change its culture is almost vacuous. Relevant here for estimation, as a *dynamic* and *interdisciplinary* model of growth, we must also consult the domain knowledge of social-cultural anthropology (cf. Guest 2013). But, regardless, one thing seems sure though, that is, disengagement with the global optima will make the job of catching-up much more difficult.

Furthermore, in terms of empirical observations, as the modulus $\{0<\Psi K_i^{(v,h,s)}<1\}$ of mathematical equation implies, our model is a *high-dimensional* and *interdisciplinary* model sensitive to deep attributes of systemic contextuality, it is therefore not surprising and indeed was predicated[26] – when our model is scaled up to the global level – that despite "complex interdependence" under conditions of globalization, *realism* (cf. Waltz 1990; Huntington 1996; Mearsheimer, 2001; Kaplan 2012) has been risin over *liberalism* (cf. Keohane & Nye 1987, 2000, 2011), both in theory and in practice in the United States vis-à-vis China, in the field of international relations, a subfield of political science.

Indeed, let me highlight here: If we are to stay close to reality, any predictive model built merely on economics without politics is inadequate, to say the least. At the global level, watch

[25] Cf. Note 8.

[26] In *Guofu Zhidao* (*or The Dao of the Wealth of Nations*) (Fu 2009), it was predicted that what has been known as "Thucydides's security dilemma" would loom large. Incidentally, *Thucydides: The Peloponnesian War* (about 5th BC), together with classical realists Machiavelli and Hobbes, was required reading in my graduate studies at Harvard University. While classical realists in IR focus on human nature and neorealists on anarchic state system, neoclassical realists take both human nature and systemic factors in their models to predict state behaviors.

out abductively at the triplet of the state, markets, and society – a sort of hype-up matrix of institutional configurations in our model – especially with respect to the relationship between China and the United States, the two largest economies in the world. In the full spectrum from economics, politics, to society, a key question is: *vector-wise*, do they continue to converge or diverge or stay where they are now, as we look forward years down the road?

2) Power of institutions

Again, unlike in the world of Euclidean geometry, in the real world where people make their livings across different countries with rich historical and cultural contents, mathematical proof in the triple of [assumption, proof, conclusion] is often understood in terms of [input data, algorithm, output data] in Diophantine approximations (cf. Cassels 1957), even though, very understandably, such proofs are known as *constructive* (cf. Bishop 1967), a term which would provoke endless philosophical debates about ontology vis-à-vis phenomenology (cf. Husserl 1982).

With that sensitivity in mind, a salient feature of the metaheuristic growth theory is that we link wealth with energy in light of the classical limit of the physical laws of thermodynamics as the kernel of our theoretical framework featuring multiple layers of causes – physical, institutional, motivational, and ideational. We then let our model run iteratively, as it were, in Hilbert space that features dot inner product (cf. Dirac 1933), and then see who is running ahead and who is lagging behind, given a certain level of energy.

Now let us do an experiment here: Take a certain amount of energy (proxied by CO_2 emissions), plug it into our model, and see how much wealth it can generate. For experimental controls, let us compare China with OECD, each having roughly the same population.[27] Combined, they represent 2.8 billion people on Earth.

The results? In China, 8 tCO_2 (currently the yearly per capita CO_2 emissions) result in about US$ 12,000 per capita; in OECD, 8 tCO_2 yield about US$ 42,000 per capita. There is a wealth differential of US$ 30,000. This is a big difference, even allowing for non-trivial margins of errors in measurement or otherwise.[28] One must explain why? Efficiency after all is central to economic analysis, and the primary justification for markets is that markets, according to Arrow and Debreu, are *efficient*. For further control in our test, now that China has become quite sophisticated in a whole range of manufacturing might that is physical, the answer has to lie in something non-physical, not directly observable, that is, institutional. Or I called it “institutional technology”.[29]

[27] China’s population is about 1.42 billion; that of OCED, representing 38 market economies, is about 1.38 billion.

[28] Comparable numbers for the US are about 14 tCO_2 per capita emissions, and per capita GDP of US$ 62,000. In terms of energy structure: in China, currently about 15% are non-fossil and the rest are fossil fuels; in the US, a 20% are non-fossil and the rest are fossil fuels.

[29] In a Global Agenda Council report of the World Economic Forum to which I contributed in writing, it said: «A key aspect of learning… ‘institutional technology’ is to draw he right balance between hierarchy and markets with the correct alignment of incentives: When do markets provides the right incentives, and when do they fail? When are private rewards aligned with social returns? How can government help align the two but not become over-reaching by turning a helping hand into a grabbing hand?» (World Economic Forum 2014).

Related here, as if to increase analytical rigor by way of sophisticated statistical difference-in-differences (DiD) approach (cf. Schwerdt & Woessmann 2020), empirical study on «Institutions, Productivity Change, and Growth», using BRICs as sample, showed that levels of institutional development (measured by corruption, property rights protection, and government effectiveness (World Bank Governance Indicators)) are a significant predictor for both growth rates and per capita GDP levels, as well as TFP levels (Akhremenko et al. 2018). This finding is broadly consistent with earlier empirical investigation into the relationship between institutional quality and capital flows (Alfaro et al. 2008). And descending further down at the business level, there is further evidence that both formal and informal institutions matter for all firms irrespective of productivity levels and technological gains; in particular, government stability, investment climate, social economic conditions, and corruption perception are essential in determining long-term productivity growth and technological changes at the firm level (Ghulam 2021).

Relevant to the power of institutions on productivity gains and economic growth, Ronald Coase (1937, 1960) was pertinent when he asked a crucial question, that is, where to draw the line between hierarchies and markets in order to minimize what he called "transactions costs". That said, his model is only 2-dimensional. By comparison, ours is a 4-dimensional model amenable to group-theoretic analysis,[30] and empirical evidence involving 2.8 billion people in the past 40 or so years resonates overwhelmingly with the Arrow-Debreu model, *vector-wise*, in Hilbert space that features dot inner product. It validates the hypothesis that markets are *efficient* – not as a pure mathematical concept, but as real-world practices contingent also upon other factors such as strong civil society and the rule of law.

And logically consistent, statistical evidence also showed that the performance of capital markets, arguably the most abstract of all markets, is contingent upon three things, reflecting network effects, that is, «marketization, globalization and the rule of law» (Fu 2000). One may take it as a mirror image of the institutional matrix of the state, markets, and society built into our model.

3) Power of ideas

Here, let us imagine an ultimate knowledge diffusion model like a pyramid. At its top lie the best ideas that humans have created, which will eventually trickle down to the bottom (cf. Rosenberg 1994). As proxies of these ideas, we have put together a dataset consisting of Fields prizes (mathematics), Nobel prizes (sciences, economics, literature, etc.), and Turing prizes (engineering). They are meant to "map", so to speak, the very frontier of human knowledge from novel ideas, new discoveries, and breakthrough technologies, or from *logos* to *rhema*, so to speak. Philosophically, that is, from *being*, *becoming*, *intuition* and *knowledge,* at the bottom of that "endless frontier" (Bush 1945) is what I see as the deep fusion of different combinations of logicism, formalism, and intuitionism.

[30] In mathematics, a scaler with no index is a tensor of zero rank, a vector with one index is a tensor of the first rank, and it is possible to extend the ideas of a tensor to three or more indexes. To analyze a high-dimensional tensor is called a group-theoretical analysis.

Our regression analysis showed that there is a strong correlation (R^2>82%) between these cutting-edge ideas and wealth (measured by GDP per capita) in a global setting. By comparison, Mankiew et al.'s empirical study – albeit an upgrade on Solow's growth accounting model – yielded a smaller R^2, i.e., less than 80%. Note in particular that while our test here looks at the "high ends" of human ideas, other empirical study on the "average case" i.e., high-school attainment rates of total population, also showed compelling results. The rate in 2010 is 74% for high-income countries; 32% for mid-level countries; and 24% for China (Rozelle et al. 2020). Note as well, these empirical results are broadly in line with our observations of the vector graphics on per capital income vis-à-vis average learning outcomes (measured by test scores of standardized, psychometrically-robust international and regional student achievement tests) shown by Our World in Data – a free educational website hosted by Oxford Martin School at the University of Oxford (https: ourworldindata.org/education).

Such being the case, for "simplicity and elegance" in "mapping the picture", so to speak, one may consider treating all other variables, including the rich institutional context, as intermediate variables between human ideas and wealth creation with varying degrees of perturbation or deviation. By taking then out to reduce noise or minimize problems of confounding and collinearity, one can ultimately demonstrate the power of ideas.[31] Here Keynes (1936) made a strong point when he said:

> [T]he ideas of economists and political philosophers, both when they are right and when they are wrong, are more powerful than is commonly understood. Indeed, the world is ruled by little else. Practical men, who believe themselves to be quite exempt from any intellectual influences, are usually the slaves of some academic scribbler of a few years back. I am sure that the power of vested interests is vastly exaggerated compared with the gradual encroachment of ideas. Not, indeed, immediately, but after a certain interval… But, sooner or later, it is ideas, not vested interests, which are dangerous for good or for evil.

In this context, it is also important to note the large energy budget the human brain consumes: For an average adult, the brain represents about 2% of the total body weight, but it accounts for about 25% of the oxygen and calories consumed by the body each day (Raichle 2002). Likewise, some of the most sophisticated AI machines such as AlphaGo (Silver et al. 2016) and ChatGPT also consume a lot of energy. AlphaGo uses about 1 megawatt, as compared to only 20 watts used by the human brain; and typically, it takes up to 10 gigawatt/hour (GWh) power to train a single LLM (large language model) such as ChatGPT-3 (Stanford University 2023). This is hardly surprising, because our brains are ultimately analog (cf. Hodgson 1988),

[31] Similarly, the Newtonian model, by assuming away the "field" (or assuming space-time as absolute not relational), achieved remarkable accuracy between theoretical predictions and empirical observations (Newton 1729). By comparison, the essence of the Einsteinian model of general relativity is to bring the "field" back, such that space-time becomes relational. As John Wheeler (1998) summarized: «Matter tells space-time how to curve and space-time tells matter how to move». In our case of complex adaptive systems research, one may want to try both out to see the differences.

and duplicating their function with a digital computer is possible but will require greater digital processing power. The issue here is *bits* (classical) or *qubits* (quantum) (cf. Georgiev 2020)?

Looking down the road, the rapid upgrading of ChatGPT and the latest developments in nuclear fusion (cf. Barbarino 2022) and in quantum computing (cf. Arute et al. 2019) may lend further support to our theory. But the risk is also huge. When the neural networks of AI systems are scaled up to 100 trillion connections, i.e., roughly as many as there are between neurons in the human brain, AI will become superhuman. Indeed, the prospect that humanity is wiped out by the technology cannot be ruled out. In order to mitigate the risk, finding a good fit between the state, markets and society – denoted as modulus $\{0<\Psi K_i^{(v,h,s)}<1\}$ in our model with deep ethical implications – is crucial and urgent. Ideally, the efforts here, regulatory in nature, ought to be global not just local, just as our fights against infectious diseases (cf. Benatar 2021), climate change (cf. Fu et al. 2023), and nuclear proliferation (cf. Coppen 2017; Smetana 2020).

4) Link between energy and wealth as a kernel of theory

Taking cues from Archimedes' Eureka (cf. Stein 1999), we go deeper in the spectrum from *solid* to *liquid* to *gas* to *plasma*, and take the parity equation or near-parity approximation between energy and wealth as the kernel of our theoretical framework.

What is kernel? Analogously, in computer science, the kernel is a core component of an operating system and serves as the interface between the system's hardware and the software processes running on it (cf. Aronszajn 1950; Berlinet 2004). Anyone who uses technologies with an operating system is working with a kernel, although often without realizing it, as the kernel has full control over everything in the system. In our model, the kernel is the interface between *pure* and *applied* mathematics and has full control over everything by linking energy with wealth within the classical limit of the physical laws of thermodynamics.

What is energy? Energy, defined in physics, is the capacity for doing work or producing heat. It may exist in potential, kinetic, thermal, electrical, chemical, biological, nuclear, or other various forms (cf. Smil 2015; Jaffe 2018). All living organisms need energy to grow and reproduce, and maintain their structure, and respond to their environment. What is unique about modern civilization is that humans have learnt how to change energy from one form to another and use it to do work, be it by human labor, machine, or otherwise.

In conventional economic terms, energy production and consumption play an important role in the economy in supply and demand. Globally, there are strong statistical associations between energy use (proxied by electricity) and wealth creation (proxied by GDP), with R^2 reaching over 90% (Burke et al. 2018; Ferguson et al. 2000); and long-term associations between energy use per capita and GDP per capita are also found to be strong (Stern 2016). The case is even more compelling, if we measure energy use by transforming all primary energy supplies into standardized toe, i.e., ton of oil equivalent defined by IEA as 107 kilocalories (41.868 gigajoules). As Figure III shows, globally as whole, the statistical association, covering the period from 1970 to 2018, between primary energy use in oil equivalent (toe) and wealth (GDP) is compelling, with R^2 reaching 99%. Indeed, as Einstein is widely quoted as saying: «Everything is energy and that's all there is to it. Match the frequency of the reality you want and you cannot help but get that reality. It can be no other way. This is not philosophy. This is physics».

This very strong, or near-parity, statistical association at the global level between energy and wealth demonstrates that the link between energy and wealth as the kernel of our theoretical framework is a valid one, as it is quite symmetrical and invariant from a systems perspective – albeit not absolutely. This is only congruent with the physical laws of thermodynamics: Whereas the first law is conservation, i.e., symmetrical, the second law is entropy, i.e., asymmetrical. The implication here is foundational, as in physics.[32] But for our purpose to explain economic growth/development, let me highlight two points: First, we can take the kernel as a benchmark devise, and launch kernel-based comparative investigation into the source or lack of it of economic growth/development. Second, unlike the "old" neoclassical model which is ultimately "lifeless" and "static", our new model is a dynamic one, reflecting disequilibria of innovation at the deep level, and as such, our model is compatible with what Schumpeter has called «the perennial gales of creative destruction» of economic growth,[33] – or the so-called "Austrian economics of competition and disequilibria" (cf. Mises 1947; Hayek 1944, 1948; Schumpeter 1942, Kirzner 1973, 1999).

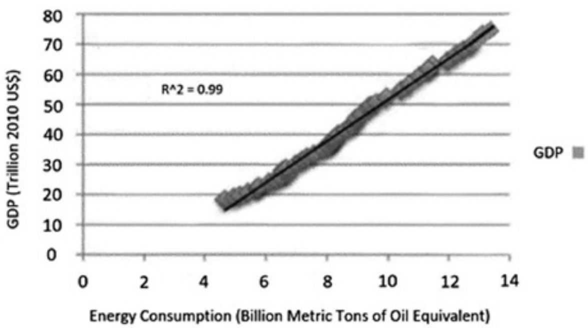

Fig. 3. Global GDP and Primary Energy Use (1970-2018).
Source: World Bank, OECD and IEA.

Indeed, at the bottom of the metaheuristic growth theory lies the *dynamism* of the confluence of three things, i.e., energy, learning, and evolution. Even though that may not be readily observable to the naked eyes, human beings survive and thrive in anti-entropic struggles via exosomatic evolution (cf. Lotka 1922; Georgescu-Roegen 1971), that is, by building instruments or technologies, including institutional technologies, such as markets and the rule of law.

And all of that takes energy. A lot of energy.

[32] While our mathematical equation is approximately symmetrical, hence the symbol ≈, the principle of conservation does not hold at the very bottom of our model. As such, our model may be taken as a mirror image at a macroscopic level of a subatomic quantum system, which is not completely symmetrical either due to weak interactions (cf. Gardner 2005). The parity violation in weak interactions, proposed first by Tsung-Dao Lee and Chen-Ning Yang (1956), was experimentally observed and validated by Chien-Shiumg Wu in 1956 (Wu 1973).

[33] Cf. note 23.

5. Conclusions

In the whole history of science from Greek philosophy there have been constant attempts to reduce seemingly complex phenomena to some simple patterns of fundamental principles and relations. The Italian scholastic philosopher Thomas Aquinas (1274), for instance, tried to capture a fundamental relationship between beauty and mathematics.[34] Modern science connecting theory and experiment really began with the work of Galileo Galilei, who systematically deployed the analytical tools of both counterfactual thought experiment and mathematics to identify laws, regularities or patterns not directly observable to the naked human eyes.

In the same spirit, this paper represents an effort to look for a simple and unified theoretical framework to explain seemingly diverse and chaotic phenomena of economic activity in the real world of non-linear, non-convex, multi-objective, and complex ecosystems.

Reduced to its bare bones, the underlying syntax of the metaheuristic growth theory is actually simple. It has only three *principal* components that can be conjugated synergistically and expressed symbolically as $\{\alpha \rightarrow \beta\psi \rightarrow \gamma^* \rightarrow \alpha\}$, like Russian Matryoshka dolls that fit one into the other. Where α stands for "synthetic *a priori*" based on logical, deductive, mathematical reason; $\beta\psi$ for "synthetic *a posteriori*" based on inductive, historical, practical reason; γ for the physical laws of thermodynamics linking wealth with energy, with * denoting *applied* as opposed to *pure* mathematics. This last point is subtle but crucial, reflecting the beauty of digital twins: whereas pure mathematicians study patterns *independent of real-world context*, we, in the field of social sciences, must study patterns, in this case, of economic growth which is necessarily *dependent of real-world context.* It is therefore critical to upgrade our cognitive toolkit to "map" the real world in high-dimensionality effectively, so to speak, such that our study remains both rigorous and relevant (cf. Scarf 1969).

As if caught in the juxtaposition of the simple and the complex at the intersection between Euclidean geometry and fractal geometry, our challenge is then to narrow the divide as closely as possible, even though we can never close it. Yet, «the mathematician's patterns», G.H. Hardy (1940) said, «like the painter's or the poet's, must be beautiful».[35] Hence *meta-heuristic* growth theory. The name implies a higher-level synthesis of "synthetic *a priori*" and "synthetic *a posteriori*", the philosophical underpinning of which, I submit, is the fusion of logicism, formalism, and intuitionism – nowadays embodied ever so subtly by the beauty of digital twins.

In the light of the above, we must have a paradigm shift from the "old" neoclassical model of Laplace-Walrasian type (cf. Wittaker 1949; Morrishima 1977) to our new model of Poin-

[34] According to him, «the science of mathematics treats its object as though it were something abstracted mentally, whereas it is not abstract in reality».

[35] The "mathematically beautiful", as I see it, does not necessarily mean mathematical truths, as proven by the unity of logic, set theory and number theory. There is a subtle difference between mathematical beauty and truth, as the beautiful would entail intuitionism, which modern mathematics does not exclude, especially in high dimensional topology. To illustrate, the "truth" of the law 2 of thermodynamics, i.e., entropy, is a statistical and not a mathematical truth, for it depends on the fact that the bodies we deal with consist of millions of molecules and that we never can get a hold of single molecules.

caré-Walrasian type (cf. Giedymin 1982). The shift entails raising dimensionality, increasing dynamism, and at the same time staying close to the real world where large numbers of small parts *interact* to produce *emergence* of a complicated whole – one of the most mysterious properties of a complex adaptive system.

Still relevant today, let me recall here: Aristotle (350 BC) said in *Metaphysics*: «In the case of all things which have several parts and in which the totality is not, as it were, a mere heap, but the whole is something besides the parts, there is a cause; for even in bodies *contact* is the cause of unity in some cases, and in others *viscosity* or some other such *quality*» (emphasis added). I say "still relevant", because in building our new model to capture the complexity of the real world, we have to take *network effect* and *emergence* seriously.

How have I gotten here? My long journey started around the start of this century. At that time, I had written my PhD dissertation. The subject-matter was about institutional impact on economic behavior. As I was revising my dissertation into a book, Robert Bates, a giant in the field of political economy at Harvard University, got me in touch with Douglass North, a Nobel Laureate in economics, who subsequently gave me generously of his time and comments. This part of my journey was acknowledged in my book-length study on *Institutions and Investments*, which was published by the University of Michigan Press in SIE series (Studies in International Economics) in 2000 (Fu 2000).

In the meantime, North told me that institutional economics was a promising research agenda, the approach was historical, but eventually it must be *formalized* [emphasis added] – a job for young generations of scholars. Honestly, I did not take his words to my heart at the time, for I was more interested in his praise of my dissertation than anything else. But looking back, that is amazing. Like a shadow that sometimes zooms in and sometimes zooms out, his words, especially the word "formalization", have been staying with me, on and off, ever since.

One may take it as another example of the power of ideas, which lies at the very basis of the metaheuristic growth theory. And let me thank him again for his intellectual prodding and inspiration.

I shall end here with a final comment on *theoria cum praxis* with respect to metaheuristic growth theory. As I said earlier, the superiority of competing theories shall be judged by the criteria of simplicity, consistency, accuracy, scope, and fruitfulness. The criterion of *fruitfulness*, in particular, ties theory with practice. During 2012-2016, I had the honor to serve as vice chair – with A. Michael Spence, another Nobel Laureate in economics, as chair – of the World Economic Forum's Global Agenda Council on New Growth Models, a high-impact policy advisory platform as its membership indicates.[36]

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[36] Members of the Council are: A. Michael Spence (Senior Fellow, Hoover Institution, Stanford University), Amanda Ellis (New Zealand Prime Minister's Special Envoy), Fu Jun (Executive Dean and Professor, School of Government, Peking University), Gavin Wilson (CEO, IFC Asset Management Company), Henry Blair (Dean, Stern School of Business, New York University), Idris Jala, (Minister, Office of the Prime Minister of Malaysia), James Wolfensohn (formerly President, The World Bank), Joyce Banda (President of Malawi), Knut Haanaes (Senior Partner, The Boston Consulting Group), Mahmoud Mohieldin (President's Special Envoy on MDGs, The World Bank), Marcello Sala (Executive Vice-Chairman, the Management Board, Intesa Sanpaolo),

In 2014, the World Economic Forum published *New Growth Models: Challenges and steps to achieving patterns of more equitable, inclusive and sustainable growth* – our council report to be distributed to key policy-makers around the world (World Economic Forum 2014).

Among other things, the report said:

> The 'new' growth models will take time to develop. They will require shifts and innovations in policies, changes in values, and new coalitions to accomplish complex transitions that require coordination. These new growth models will contain essential ingredients of the 'old' models: an open global economy, specialization as a function of stage of development, innovation and competition, and high levels of investment. But there will be new elements.

In particular, apart from "natural capital base", the report emphasized the importance of "institutional technology" and "institutional learning and innovation", as well as the role of "entrepreneurship" and "entrepreneurial ecosystems".

As one can see, these old and new elements are all consistent with the key ingredients of the metaheuristic growth theory that I have outlined above but formulated mathematically with a new perspective in high dimensionality. Let me recap: amenable to group-theoretic analysis, our new model is a high-dimensional, complex, and dynamic model, with its kernel linking wealth and energy ultimately constrained within the classical limit of the physical laws of thermodynamics. As such, *theoria cum praxis*, while the model is highly mathematical, it is intrinsically embedded in reality, and is greater in both predictive and prescriptive power than the canonical model of classical or neoclassical economics.

It is a paradigm shift.

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Martin Wolf (Associate Editor and Chief Economics Commentator, Financial Times), Masahiro Kawai (Dean and CEO, Asian Development Bank Institute), Mustafa Abdel-Wadood, Chairman, the Management Executive Committee, The Abraaj Group), Pier Carlo Padoan, Deputy Secretary-General and Chief Economist, OECD), Simon Zadek (Senior Fellow, Global Green Growth Institute), Trevor Manuel (Minister, the National Planning Commission, South Africa), Victor Fung (Chairman, Fung Group), and Yu Yongding (Senior Fellow, Chinese Academy of Social Sciences).

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